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A Comparative Review of Techniques for the Detection of Fingerprints' Orientation

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Abstract— Accurate fingerprint recognition is a function of accurate detection of fingerprint features, basically, fingerprint ridge orientation. To this end, different techniques have been proposed and implemented. To keep pace with the increasing number of techniques being developed, this paper comparatively and extensively, reviewed existing techniques for modeling fingerprint ridge orientation. Recent research papers on the techniques for the estimation of fingerprint ridge orientation were evaluated under the two major categories of global and local techniques respectively. Three techniques were selected and experimentally tested and compared by determining their computation time, accuracy to represent the ridge orientation field, and robustness to noise using the FVC2000 DB2a database in a MATLAB R12b programming environment.

Keywords— Fingerprint, Ridge Orientation, Orientation Map, Local ridge orientation estimation, Global modeling.

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1 INTRODUCTION

THE need for accurate fingerprint orientation map estimation has deepened by the great demands of fingerprint biometric in forensic applications in law courts, access controls in both private and commercial uses, etc. Fingerprint feature extraction techniques have been proposed and developed. Fig. 1, shows feature extraction; an essential module of fingerprint recognition systems. Most feature extraction techniques are based on the striking topological characteristics of fingerprints. A striking topological characteristic of fingerprints is the highly parallel oriented pattern, fingerprint orientation map, shown in Fig. 2. It is not only useful for fingerprint enhancement but for its recognition, classification, matching, indexing, and for other features extraction operations. Three kinds of features are usually in use in literature, namely: global features, local features, and fine features (sweat pores of intra-ridge details) [1]. They are categorized as level 1, level 2, and level 3 [2]. Level 1 features are the macro details, namely ridge flow map (orientation field), ridge quality map, ridge wavelength map, and pattern type. Orientation field is key to the determination of levels 2, and finer detailed discriminatory level 3 features. Features gotten from orientation field are more robust to image noise. Accurate fingerprint orientation map estimation has proven to be a no-easy-work. Hence, the proposition of several techniques establishing different solutions. These techniques have grown in population and are still growing. Motivated by this development, this paper is aimed at creating a better understanding of the developed techniques, thereby, opening up new ways to the development of accurate, reliable and robust fingerprint biometric systems. The remainder of the paper is arranged as follows:

section 2 is an overview of local and global orientation map estimation techniques.

section 3 reviewed some recently proposed techniques in research literature.

Section 4 experimentally compared some selected techniques; gradient-based method – a local approach – against Legendre polynomial and Fourier series 2D expansion models – two global approaches.

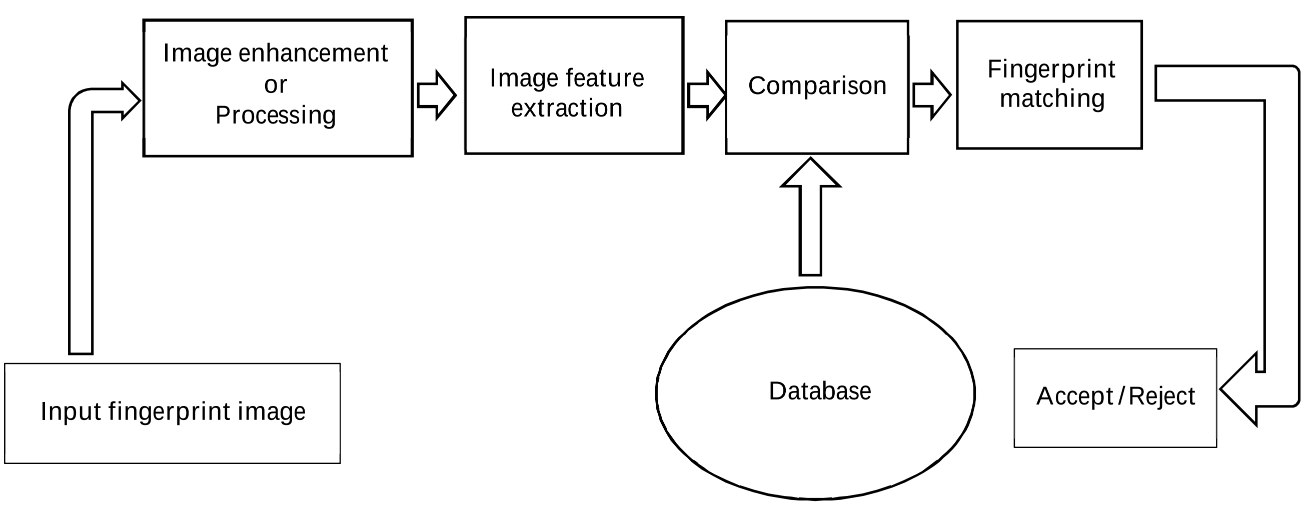

Figure 1. Generalized Block Diagram of Fingerprint Recognition System.

Figure 2. Parallel Oriented pattern of fingerprint (image: FVC DB2_A_2000).

2 Overview of Local and Global Orientation Field Estimation Techniques

There are majorly two groups of fingerprint orientation estimation techniques in literature; local, and global techniques.

A. Local Ridge Orientation Estimation Techniques

Local ridge orientation estimation techniques are based on the assumption that the orientation at a pixel position is the direction for which the signal changes the least. Some of them compute gray consistency in some chosen directions at each separate pixel and take the orientation of the most consistent as the ridge orientation of the pixel involved while majority of them compute the average gradient vector direction of the pixel in the local block noting that the gradient vector direction of an edge pixel is orthogonal to the ridge orientation of the local block with good structure. If the local block is not of good structure, the gotten orientation will not represent the true orientation of the block. Therefore, they tend to generate noisy orientation fields known as the coarse orientation. They are classified as spatial domain based algorithms e.g., gradient-based methods [3], [4], and frequency space based algorithms [5], [6] e.g., the Matched-filter approaches [7]. Unfortunately, determination of ridge orientation becomes more difficult as image quality degrades. Therefore, most local techniques do not necessarily compute the ridge orientation at each pixel, but over a square-meshed grid [8] to reduce the effect of image quality. The gradient-based approach is believed to be more accurate and computationally efficient, though, noise sensitive than other methods mainly due to a limited number of fixed possible orientations. As a result, gradient-based methods are the most common approach for local orientation estimation.

B. Global Ridge Orientation Estimation Techniques

Most advanced orientation field estimation methods are global methods, which rely on the smooth regularity of orientation values around singular points [9]. The realization of fingerprint orientation field via local ridge orientation techniques produce coarse orientation which is noise sensitive. To obtain orientation image with less noise effect, usually, the coarse ridge orientation gotten from local estimation techniques are smoothened [10]. Global techniques are the trend followed by recent researches in ridge orientation estimation, as they tend to produce a more accurate result, though, computationally intensive and waste time. They are (a) local smoothness assumption based techniques, (b) dictionary look-up based techniques, and (c) model based. The local smoothness assumption methods include low-pass filtering based methods [11] being the commonest, and Markov random field methods [12]. The dictionary look- up include the localized dictionary look-up [13]. Model based methods are zero-pole model [14], point charge model [15], phase portrait model [16], Fourier series model [17], Legendre polynomial model [18], etc.

3 Review of Local and Global Orientation Field Estimation Techniques

I. Local Techniques

The gradient-based technique being the commonest of the local orientation techniques is based on the orthogonal nature of ridge orientation. It considers the orientation field as a collection of the partial derivatives of the gradients – orientation at the pixel level. It proceeds by developing an estimator for the local flow direction – the direction with slow intensity variation as a result of the underlying anisotropic process. Most local ridge orientation estimation techniques tend to process fingerprints ridge orientation block-wise in order to reduce computation and storage complexity while making the technique efficient [11], [19]. The gradient vector is computed by taking the partial derivatives of gray values at every pixel, and averaging the squared gradients in a local neighborhood gives the ridge orientation. As described in [20], the dominant ridge orientation in a square block is computed by determining the peak magnitude spectrum of the image.

This technique produces coarse ridge orientation, it is quite sensitive to noise and requires post-processing to reduce the noise effect. Due to its advantages in describing local orientation field of a fingerprint, several researchers have developed different improvements to it, and most global techniques adopt it in their scheme.

II. Global Techniques

Global techniques rely on the smooth regularity of orientation values around singular points [9]. Earlier generalized approaches were based on applying a global model on the estimated local orientation information. Research has shown that dependence on local information for global modeling is one of the causes of the failures of most available global models. This arises because local techniques are unsatisfactory in noisy images. The current trend aims at avoiding the dependence on local information while more attention is paid to preserving high curvature areas. The majority of this category are model-based. Examples are 2D Fourier series expansion method [17], and Legendre polynomials approach [21]. They possess better orthogonal property. A number of issues affect their performances which bother on their generality, their lack of constraints on the valid range of parameters to be used, computation time, cost, and complexity. Additionally, most of them depend on the explicit determination of singular points which in turn is not a trivial issue to contend with as determination of singular points is another problem on its own.

Table 1. A comparison of local fingerprints orientation field detection techniques.

TITLE	TECHNIQUE AND OBJECTIVE	RESULTS/DATABASE USED	ACHIEVEMENTS AND LIMITATIONS
A Multi-Scale Approach to Directional Field Estimation [22]	Use of multiple scales. Robust directional field estimation.	Satisfactory in scratchy singular point regions.	Improved directional field estimation at scratchy singular points regions. Heavy computation complexity.
A Gradient-Based Weighted Averaging Method for Estimation of Fingerprint Orientation Fields [23]	Weighted averaging technique. To accurately estimate ridge orientation.	Better noise resistant. FVC2000	A more robust gradient-based ridge orientation estimation. Heavy computation complexity.
Ridge Orientation Estimation and Verification Algorithm for Fingerprint Enhancement [24]	Hybrid enhancement technique. To reliably estimate fingerprint ridge flows.	Improved quality of a fingerprint. NIST-4	Improved quality of fingerprint image. High involvement of particular isotropic and anisotropic filters.
Fingerprint Ridge Orientation Estimation Based On Neural Network [25]	Neural network approach. To accurately estimate ridge orientation.	Outperformed gradient-based methods. FVC2000 DB1	Accurate ridge orientation estimation. Sensitive to noise.
Enhanced gradient-based algorithm for the estimation of fingerprint orientation fields [26]	Gradient based method. To improve ridge orientation estimation using the gradient-based method.	Improved ridge orientation extraction. Less computation cost. FVC2000	Better noise resistance. Modest computation time. Noise sensitive.
Fingerprint Orientation Field Estimation Using Ridge Projection [27]	Use of neural pulses of pulse coupled neural network (PCNN). To estimate ridge orientation faster.	Improved ridge orientation detection at reduced processing time. FVC2004	Improved fingerprint orientation field estimation. Fast processing speed. Low computation cost. Not so robust to noise.
Robust Orientation Field Estimation and Extrapolation Using Semi-Local Line Sensors [28]	Line Sensor Operator approach. To robustly extract fingerprint orientation field.	Better performance than gradient-based and multi-scale approaches. FVC 2000, 2002, and 2004.	Robust orientation field estimation. High computation cost. Slow computation.
The Pixel Alignment Based Algorithm for Continuous Orientation Field Estimation [29]	Pixel Alignment Approach. To accurately estimate continuous orientation values.	Better performance than the gradient-based model. NIST special database, FVC, and Live-scan.	More efficient with reduced Equal Error Rate (EER). Prone to noise. High computation cost.
A Reliable Fingerprint Orientation Estimation Algorithm [30]	Interpolation approach via separation of image blocks. To extract ridge orientation.	More accurate than gradient-based methods. NIST-4	Enhanced fingerprint orientation estimation. Not robust to noise.

Table 2. A comparison of global fingerprint orientation field detection technique

TITLE	TECHNIQUE AND OBJECTIVE	RESULTS/DATABASE USED	ACHIEVEMENTS AND LIMITATIONS
Nonlinear Phase Portrait Modeling of Fingerprint Orientation [31]	Model-based technique. To accurately represent fingerprint orientation field.	Ability to model ridge orientations of fingerprints. Able to model multiple singular points. NIST special database 4.	It models all types of fingerprint orientations using a single model. High computation cost. Heavy dependence on singular points detection.
A Model-Based Method for the Computation of Fingerprints' Orientation Field [32]	Combination technique. To achieve better performance.	Accurate estimation of ridge orientation. NIST 14, FVC2000 and THU.	More robust to noise than gradient-based and filter-based algorithms. Dependent on singular points detection. High computation cost.
A Fingerprint Orientation Model Based On 2D Fourier Expansion (FOMFE) and Its Application to Singular-Point Detection and Fingerprint Indexing [17]	2D Fourier series expansion technique. To accurately represent fingerprint global features.	Improved feature extraction of all types of fingerprints. FVC2002 DB1a	Independent of singular points detection. Low computation cost. Uses heuristic approach to determine model parameters. Not robust to noise.
Curvature Preserving Fingerprint Ridge Orientation Smoothing Using Legendre Polynomials [33]	Use of Legendre polynomials. To improve ridge orientation smoothing.	Improved orientation field smoothing. FVC2004	Improved ridge orientation smoothing at high curvature areas. Not robust to noise. Computationally expensive.
Global Models for the Orientation Field of Fingerprints: An Approach Based on Quadratic Differentials [34]	Quadratic differential. To accurately estimate ridge orientation.	Ability to model all fingerprint classes with fewer available singularities. NIST special database 4.	Description of ridge orientation using fewer parameters. Heavily dependent on singularities. Lacks flexibility.
A Chebyshev/Legendre Polynomial Interpolation Approach for Fingerprint Orientation Estimation Smoothing and Prediction [10]	Interpolation technique. To effectively estimate ridge orientation.	Performs better than other methods. FVC2000 DB2 and FVC2004 DB3 and DB4 databases.	Better ridge orientation estimation. It is computationally complex.
Modeling Fingerprint Ridge Orientation using Legendre Polynomials [18]	Combination technique. To smoothen orientation data while preserving high curvature areas.	Improved singular point detection and matching rates. FVC2004 DB3a and FVC2006 DB2a.	Very compact representation of the orientation field. Computationally complex.
Invariant Representation of Orientation Fields for Fingerprint Indexing. [35]	Polar complex moment (PCM). Rotation invariant estimation of orientation field.	Improved rotation invariant ridge orientation estimation. NIST DB4 and FVC2002.	Robust ridge orientation extraction. Computationally complex.
Fingerprint Orientation Map Based on Wave Atoms Transform [36]	Wave atoms transform. To uniquely extract fingerprint ridge orientation.	Performs better than gradient-based methods. FVC 2004 database.	Improved ridge orientation extraction. It is not robust to noise. Laborious and takes time to compute.

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4 Experimental Comparison of Gradient based Techniques, Legendre Polynomial, and 2D Fourier series Expansion based Techniques

This section takes into consideration a detailed experimental review of a local approach based on gradient method proposed in [11] which is adopted by different researchers for the computation of coarse fingerprint ridge orientation, and two global approaches namely Fingerprint Orientation Modeling based on 2D Fourier Expansion (FOMFE) proposed in [17], and Legendre polynomial approach introduced in [21]. All the algorithms were implemented using MATLAB R12b environment on a 4 GB RAM, 2.4MH speed Intel Core i5 processor. The experiment is carried out using Fingerprint Verification Competition FVC2000 DB2a database [37]. This database contains fingerprint images that are captured by a capacitive sensor with a resolution of 500 pixels per inch. It has 100 untrained individuals enrolled, each with eight prints of the same finger adding up to a total of 800 fingerprints.

A. Gradient-based approach

The five images in figure 3 labeled a, b, c, d, and e are selected from five different enrolments of the said database. It can be seen that the images have varying degrees of noises. The local ridge orientation images obtained are as shown in figure 4.

Figure 3. Original grayscale images (from FVC2000 DB2a).

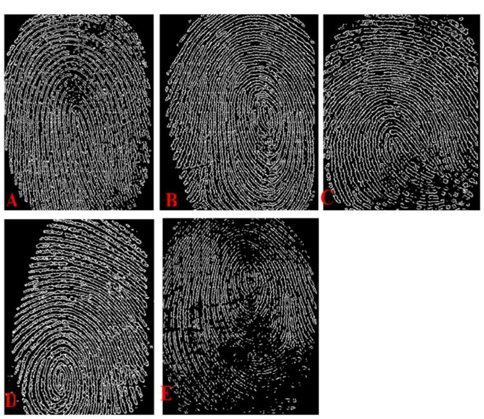

Figure 4. Local ridge orientation images of grayscale images of figure 3 using gradient-based technique.

The coarse ridge orientation images of figure 4 above show the effect of noise on the orientation images. Notwithstanding its instability in noisy images, this approach still represents the ridge directions to a reasonable extent as can be seen in figure 5 below.

Figure 5. A superimposition of the original grayscale image with its coarse orientation image.

As seen in the superimposition of the coarse ridge orientation image and the original images of figure 3, the regions with high noise effect are those regions where the ridge lines are not clearly represented. In Figure 5, images labeled A, C, and E show, more prominently, the effects of noise. This can be seen more clearly with the circled marks as depicted in the figure.

B. Fourier Series Expansion based approach

This implementation involves mainly two steps; first, the computation of coarse ridge orientation and its training, and second, the reconstruction of the image by fitting the modeling parameters. The coarse ridge orientation serves as the input for the data fitting stage. Ridge orientation tends to be discontinuous when rotated over 180°. To overcome this discontinuity, the orientation field is mapped into a new vector field where its elements are represented as 2-dimensional vectors. With θ as the orientation angle, the 2D vector is given as $\boldsymbol{v} = (v_c, v_s)$. Where v_c and v_s are phase functions of $\cos 2\theta$ and $\sin 2\theta$ respectively [3]. Here, global ridge orientation is represented using a dynamic system of differential equations,

$$f(x,y) = \sum_{m=0}^{k}\sum_{n=0}^{k}\varphi(mvx, nwy; \beta_{mn}) + \varepsilon(x,y), \quad (1)$$

where m and $n \in N$; the fundamental frequencies v and ω are orthogonal to the x and y axes.

$$\begin{aligned}\varphi(mvx, nwy; \beta_{mn}) &= \lambda_{mn}[a_{mn}\cos(mvx)\cos(nwy) \\ &+ b_{mn}\sin(mvx)\cos(nwy) \\ &+ c_{mn}\cos(mvx)\sin(nwy) \\ &+ d_{mn}\sin(mvx)\sin(nwy)\,, \end{aligned} \quad (2)$$

Where β_{mn} the parameter composed of Fourier coefficients $\{a_{mn}, b_{mn}, c_{mn}, d_{mn}\}$, and λ_{mn} is a scalar constant. The modeling function is given as

$$\dot{X} = \boldsymbol{f}(X), \quad (3)$$

$\boldsymbol{f}$, is the mapping function.

The number of parameters with the corresponding order is as contained in table 3 below.

TABLE 3. THE PARAMETERS FOR A GIVEN ORDER OF POLYNOMIAL FOR FOMFE AND LEGENDRE METHODS

	order	FOMFE	Legendre
Number of parameters	2	50	12
	3	98	20
	4	162	30
	5	242	42
	6	338	56
	7	450	72
	8	578	90
	9	722	110
	10	882	132

Figure 6 is the orientation images using Fourier series expansion model corresponding to the input images of figure 3.

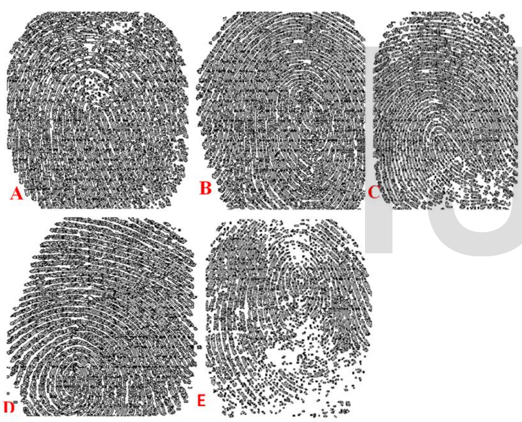

Fig. 6. Ridge orientation image obtained from Fourier series expansion (FOMFE) as the basis function.

Fig. 7. A superimposition of the ridge orientation images of FOMFE method on the original gray-scale images of figure 5.

From the images of figure 6, it can be seen that the ridge and valley lines are more visible compared to the images of figure 4. The improvement in this regard is due to the inherent orthogonal property of trigonometric function used as the basis function in the model. Figure 7, is a superimposition of the original grayscale images of the database used and the ridge orientation images of the Fourier series expansion model. It shows how closely the approach is able to represent the original images.

C. Legendre polynomial based approach

Since ridge orientation is orthogonal to the gradients, a modeling function with good orthogonal property tends to produce better ridge orientation. The choice of orthogonal polynomials arises as they do not result in ill-conditioned equation systems that get worse as the orders of the polynomial go higher [21]. This assures that their round-off errors get small and makes the optimization approach stable, even when the data sets get larger. They also possess the advantage that their discretization error is minimized and the resulting parameter space is Euclidean [21]. Implementing the technique, a combination of the basis functions having optimal 9 order polynomials are used.

At every point, the following equation is evaluated:

$$f(x, y) \approx \sum_{j=0}^{n} a_j \emptyset_j(x, y) \quad (4).$$

$$\Phi(x_i) = [\,\phi_0(\boldsymbol{x}), \phi_1(\boldsymbol{x}), \ldots, \phi_n(\boldsymbol{x})] \quad (5)$$

$\Phi(x_i)$ is the row vector having the set of basis as the left-hand side of equation (5) evaluated for a given coordinate $\boldsymbol{x} = (x, y)$.

Figure 8, below is ridge orientation images using Legendre polynomial as the basis function.

Figure 8. Ridge orientation images using Legendre polynomial as the modeling function.

Figure 9. Original images superimposed with their corresponding orientation images using Legendre polynomial.

5 Discussion and Experimental Results

Each orientation field images of the three approaches are superimposed on the corresponding original gray-scale images labeled a – e with varying qualities. From figures 5, 7, 9, and 3, it is seen that the three techniques are able to redraw the fingerprint image ridges reasonably, except for singular point regions. It can be seen from figures 7 and 9 as well as from the very poor image qualities of the last row of figure 3, that Fourier series expansion model is able to redraw the smeared ridge structures that can hardly be distinguished by considering the orientation images of figure 4 and its superimposed version of figure 5 gotten from the conventional gradient-based techniques. Figures 9, and 7 show that the Legendre polynomial based model gave a better result than the Fourier series expansion model and the Gradient-based technique. Hence, orientation field extraction based on global techniques are more reliable and accurate. Computation-wise, the global approaches are more computationally demanding compared to the local approach. From table 4, it is shown that gradient-based method, a local technique, took 7.789058 seconds to generate the orientation image while the two global approaches took 14.235022 seconds on the average, with the Legendre polynomial method taking 13.471316 seconds while the Fourier series expansion model took 14.998729 seconds to generate the orientation images. This is due to the number of parameters to be computed while using the global techniques, unlike the gradient-based approach that involves none. The Legendre polynomial approach possesses finer graduation of the number of parameters needed for the generation of the orientation images compared to Fourier series expansion model, and as such, it is fast to compute and evaluate.

Table 4. Comparison of Computation Time of the Techniques

Technique	Computation time in seconds
Gradient-based	7.789058
2D Fourier series expansion model	14.998729
Legendre Polynomial model	13.471316

6 Conclusion

This paper comparatively and extensively reviewed commonly available techniques in the research literature for modeling fingerprint ridge orientation. Experimental comparison between the gradient based approach – a local technique – and the Fourier 2D modeling approach with Legendre polynomial based approach were carried out to summarize the observations of the evaluations. The comparison was done through the determination of computation time, robustness, and the accuracy of the techniques using MATLAB R12b programming environment. The result of the comparison shows that the local techniques are fast to compute, but give rise to coarse ridge orientation estimation. On the other hand, the global techniques are more robust to noise and efficient in the global description of the ridge orientation at the same time preserve areas of high curvature. Hence, orientation field extraction based on global techniques are more reliable and accurate. Computation-wise, the global approaches are more computationally demanding compared to the local approach.

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